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# SUGGESTED SOLUTION

## CA FOUNDATION

**SUBJECT-** MATHS, LOGICAL REASONING & STATS

## Test Code – CFN 9272

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**1. B** **[Ans.: B]**

**2.** $f(x) = x^k$ $\therefore f'(x) = k \cdot x^{k-1}$

$\therefore f'(1) = k \cdot (1)^{k-1} = k = 10$ **[Ans.: A]**

**3.** $\int x(x^2+4)^5\,dx = \frac{1}{2}\int (x^2+4)^5 \cdot (2x)dx = \frac{1}{2}\frac{(x^2+4)^6}{6} + c$

$= \frac{1}{12}(x^2+4)^6 + c$ **[Ans.: D]**

**4.** $f(x) = 100x$, $f^{-1}(x) = ?$

$\therefore y = 100x$

$\therefore x = \frac{y}{100}$ $\therefore f^{-1}(y) = \frac{y}{100}$ $\therefore f^{-1}(x) = \frac{x}{100}$

**[Ans.: A]**

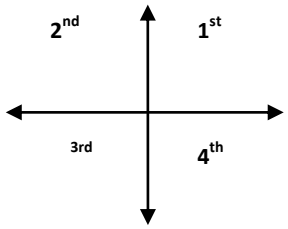
**5.** $y = 4x^3 - 7x^4$

$\therefore \frac{dy}{dx} = 12x^2 - 28x^3 = 2x(6x - 14x^2) = 2x(-14x^2 + 6x)$ **[Ans.: B]**

**6.** $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx = \int \left(x^{1/2} + x^{-1/2}\right)dx = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + c$

$= \frac{2x^{3/2}}{3} + 2x^{1/2} + c = 2x^{1/2}\left(\frac{1}{3}x + 1\right) + c$ **[Ans.: B]**

**7.** In 3rd quadrant, $x < 0$, $y < 0$ **[Ans.:A]**




**8.** $x^2 + y^2 = a^2$ $\therefore 2x + 2y\frac{dy}{dx} = 0$ $\therefore 2y\frac{dy}{dx} = -2x$

$\therefore y\frac{dy}{dx} = -x$ $\therefore \frac{dy}{dx} = -\frac{x}{y}$ **[Ans.: C]**

**9.** $\int (4x^3 + 3x^2 - 2x + 5)dx = 4\left(\frac{x^4}{4}\right) + 3\left(\frac{x^3}{3}\right) - 2\left(\frac{x^2}{2}\right) + 5x + c$

$= x^4 + x^3 - x^2 + 5x + c$ **[Ans.:C]**

**10.** Range = {3, 5, 7} **[Ans.: B]**

= set of values of y

**11.** $x = at^3$, $y = \frac{a}{t^2} = a.t^{-2}$ Now $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2at^{-3}}{3at^2}$ **[Ans.: D]**

$\frac{dx}{dt} = 3at^2$, $\frac{dy}{dt} = -2at^{-3}$ $= \frac{-2}{3t^5}$

**12.** $f(x) = 10x - 7$

$\therefore y = 10x - 7$

$\therefore 10x = y + 7$

$\therefore x = \frac{y+7}{10}$

$\therefore f^{-1}(y) = \frac{y+7}{10}$

$\therefore f^{-1}(x) = \frac{x+7}{10}$

$\therefore g(x) = \frac{x+7}{10}$

**[Ans.: C]**

**13.** $A \Delta B = (A - B) \cup (B - A) = \{1, 2, 4\} \cup \{5, 7\}$

$= \{1, 2, 4, 5, 7\}$ [Ans.: A]

**14.** $y = x^2 . \log x$

$\therefore \frac{dy}{dx} = x^2 \left(\frac{1}{x}\right) + \log x \, (2x) = x + 2x \log x$

$= x (1 + 2 \log x)$ **[Ans.: C]**

**15.** $\int x(x-1)^{-1} (2x+1)^{-1} \, dx = \int \frac{x}{(x-1)(2x+1)} dx$

$$\frac{x}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$$

$\therefore x = A (2x + 1) + B (x - 1)$

$x = 1 \quad \therefore 1 = A (3) + 0 \qquad \therefore A = 1/3$

$x = -1/2 \qquad \therefore -\frac{1}{2} B (-3/2) \qquad \therefore B = 1/3$

$\therefore \int \frac{x}{(x-1)(2x+1)} dx = \frac{1}{3} \int \frac{1}{x-1} dx + \frac{1}{3} \int \frac{1}{2x+1} dx$

$= \frac{1}{3} \log(x - 1) + \frac{1}{3} log \frac{(2x+1)}{2} + c$

$= \frac{1}{3} \left[\log(x - 1) + \left(\frac{1}{2}\right) log (2x + 1)\right] + c$ **[Ans.: C]**

**16.** $y = e^{3x}$

$\therefore y' = 3e^{3x}$

$\therefore y'' = 9e^{3x}$ **[Ans.: D]**

**17.** $\int log (a^x) dx = \int x \log a \, dx = \log a \int x \, dx$

$= \log a \left(\frac{x^2}{2}\right) + c$ **[Ans.: A]**

**18. [Ans.: D]**

**19.** $x^m\ y^n = (x + y)^{m+n}$

Taking log on both the sides

$m \log x + n \log y = (m + n) \log (x + y)$
Differentiating w.r.t.x.

$$\mathbf{m.}\ \frac{1}{x} + n.\frac{1}{y}\frac{dy}{dx} = m + n\ \frac{1}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$

$$\frac{\boldsymbol{m}}{\boldsymbol{x}} + \frac{\boldsymbol{n}}{\boldsymbol{y}}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\frac{\boldsymbol{m}}{\boldsymbol{x}} + \frac{\boldsymbol{n}}{\boldsymbol{y}}\frac{dy}{dx} = \frac{m+n}{x+y} + \frac{m+n}{x+y}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\frac{dy}{dx}\left(\frac{nx+ny-my-ny}{y(x+y)}\right) = \frac{mx+nx-mx-my}{x(x+y)}$$

$$\frac{dy}{dx} = \frac{y(nx-my)}{x(nx-my)}$$

$$\frac{dy}{dx} = \frac{y}{x}$$ **[Ans.: B]**

**20.** $\int xe^x(x+1)^{-2}\,dx = \int \frac{x}{(x+1)^2}e^x dx = \int \frac{x+1-1}{(x+1)^2}\,e^x\,dx$
$= \int \left[\frac{1}{x+1} \cdot \frac{-1}{(x+1)^2}\right]\,e^x\,dx$

$= \frac{1}{x+1} \cdot e^x + c\ [\because \int f(x) + f'(x)]e^x dx = f(x)e^x + c]$

$= e^x\,(x+1)^{-1} + c$ **[Ans.: B]**

**21.** $f(x) = 2x + h$

$f(x + h) = 2(x + h) + h = 2x + 2h + h = 2x + 3h$

Now, $f(x + h) - 2\ .f(x) = 2x + 3h - 2(2x + h)$

$= 2x + 3h - 4x - 2h$

$= h - 2x$ **[Ans.: A]**

**22.** $\int e^x(x^2 + 2x)dx) = x^2 \cdot e^x + c$

$[\because \int e^x[f(x) + f'(x)]dx = f(x)e^x + c]$ **[Ans.: A]**

**23.** **[Ans.: D]**

**24.** $y = \log x^x$

$\therefore = x \cdot \log x$

$\therefore \frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1) = 1 + \log x$

$= \log x + 1$

$= \log x + \log e \qquad (\because \log e = 1)$

$= \log (x\,e)$

$= \log ex$ **[Ans.: A]**

**25.** $\int (x^2+1)^{-3} \cdot x^3\, dx = \int \frac{x^3}{(x^2+1)^3} dx = \int \frac{x^2}{(x^2+1)^3} \cdot x\, dx$

Taking $y = x^2 + 1 \qquad \therefore dy = 2x\, dx \quad \therefore x\, dx = \frac{dy}{2}$

$\therefore x^2 = y - 1$

$= \int \frac{y-1}{y^3} \cdot \frac{dy}{2} = \frac{1}{2}\int \left(\frac{y}{y^3} - \frac{1}{y^3}\right) dy$

$= \frac{1}{2}\int (y^{-2} - y^{-3}) dy = \frac{1}{2}\left[\frac{y^{-1}}{-1} - \frac{y^{-2}}{-2}\right] + c$

$= \frac{1}{2}\left[-\frac{1}{y} + \frac{1}{2y^2}\right] + c = \frac{1}{2}\left[\frac{-2y+1}{2y^2}\right] + c$

$= \frac{-2(x^2+1)+1}{4(x^2+1)^2} + c = \frac{-2x^2-2+1}{4(x^2+1)^2} + c$

$= \frac{-2x^2-1}{4(x^2+1)^2} + c = -\left(\frac{1}{4}\right)(2x^2+1)/(x^2+1)^2 + c$ **[Ans.: D]**

**26.** $f(x) = x \qquad \therefore fofof(x)$

$= f(fof(x))$

$= f[f(x)] = f(x) = x$ **[Ans.: A]**

**27.** $y = \left(x^{1/3} - x^{-1/3}\right)^3$

$\therefore y = \left(x^{1/3}\right)^3 - 3\left(x^{1/3}\right)^2\left(x^{-1/3}\right) + 3\left(x^{1/3}\right)\left(x^{-1/3}\right)^2 - \left(x^{-1/3}\right)^3$

$\therefore y = x - 3\,x^{2/3}\,x^{-1/3} + 3x^{1/3}\,x^{-2/3} - x^{-1}$

$\therefore y = x - 3x^{1/3} + 3x^{-1/3} - x^{-1}$

$\therefore \frac{dy}{dx} = 1 - x^{-2/3} - x^{-4/3} + x^{-2} = 1 + x^{-2} - x^{-2/3} - x^{-4/3}$ **[Ans.: D]**

**28.** $\int (e^x + e^e)dx = e^x + e^e \cdot x + c$ **[Ans.: A]**

**29.** $y = e^{a\log x} + e^{x\log a}$

$\therefore y = e^{\log x^a} + e^{\log a^x}$

$\therefore y = x^a + a^x$ $[\because e^{\log f(x)} = f(x)]$

$\therefore \frac{dy}{dx} = a \cdot x^{a-1} + a^x \cdot \log a$ **[Ans.: B]**

**30.** $\int \frac{(5+\log x)^2}{x} dx = \int (5 + \log x)^2 \cdot \frac{1}{x} dx$

$= \frac{(5+\log x)^3}{3} + c$ **[Ans.: A]**

**31.** $A - B = A \cap B'$ **[Ans.: C]**

**32.** $f(x) = e^{ax^2 + bx + c}$

$\therefore f'(x) = e^{ax^2+bx+c}(2ax + b)$ **[Ans.: B]**

**33.** $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx = \int e^x \left[\frac{1}{x} + \frac{-1}{x^2}\right] dx = e^x \cdot \frac{1}{x} + c$

$[\because \int e^x (f(x) + f'(x)]\, dx = e^x \cdot f(x) + c]$

$= \frac{e^x}{x} + c$ **[Ans.: C]**

**34.** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 20{,}000 + 32{,}000 - 6{,}000$

$= 46{,}000$

$n(A' \cap B') = \text{Total} - n(A \cup B)$

$= 60{,}000 - 46{,}000 = 14{,}000$ **[Ans.:A]**

**35.** $y = \log (3x + 4)^{1/2} = \frac{1}{2} \log (3x + 4)$

$\therefore \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{3x+4} \cdot (3) = \frac{3}{2}(3x + 4)^{-1}$

$\therefore \frac{d^2y}{dx^2} = -\frac{3}{2}(3x + 4)^{-2} \cdot (3) = -\frac{9}{2}(3x + 4)^{-2}$

$\therefore \frac{d^3y}{dx^2} = 9 (3x + 4)^{-3} \cdot (3) = 27 (3x + 4)^{-3}$

$\therefore \frac{d^4y}{dx^4} = -81 (3x + 4)^{-4} \cdot (3) = -243 (3x + 4)^{-4}$ **[Ans.: B]**

**36.** $\int_0^1 (2x+1)^2 dx = \left[\frac{(2x+1)^3}{3(2)}\right]^1 = \frac{(3)^3}{6} - \frac{(1)^3}{6}$

$= \frac{27-1}{6} = \frac{26}{6}$ **[Ans.: C]**

**37.** $x = at^2$ $y = 2at$ Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$

$\therefore \frac{dx}{dt} = 2at$ $\frac{dy}{dt} = 2a$ **[Ans.: A]**

**38.** $\int (3 - 2x - x^4)dx = 3x - 2\left(\frac{x^2}{2}\right) - \left(\frac{x^5}{5}\right) + c$

$= 3x - x^2 - \left(\frac{x^5}{5}\right) + c$ **[Ans.: B]**

**39.** $f(x) = log_{10}^x$

$\therefore y = log_{10}^x$ $\therefore x = 10^y$ $\therefore f^{-1}(y) = 10^y$

$\therefore f^{-1}(x) = 10^x$ **[Ans.: A]**

**40.** $y = \sqrt{x + \sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x\sqrt{x}}}\left[1 + \frac{1}{2\sqrt{x}}\right]$ **[Ans.: C]**

**41.** $\int_1^2 \frac{x}{x^2+1}\, dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx = \left[\frac{1}{2}\log\left(x^2+1\right)\right]_1^2$

$= \frac{1}{2}\log(5) - \frac{1}{2}\log(2) = \frac{1}{2} log_e\left(\frac{5}{2}\right)$ **[Ans.: B]**

**42.** $B - A = \{6, 3\}$ **[Ans.: C]**

**43.** **[Ans.: C]**

**44.** $y = \frac{5x^4-6x^2-7x+8}{5x-6}$

$\therefore \frac{dy}{dx} = \frac{(5x-6)(20x^3-12x-7)-(5x^4-6x^2-7x+8)(5)}{(5x-6)^2}$

$= \frac{100x^4-60x^2-35x-120x^3+72x+42-25x^4+30x^2+35x-40}{(5x-6)^2}$

$= (75x^4 - 120x^3 - 30x^2 + 72x + 2)(5x - 6)^{-2}$ **[Ans.: C]**

**45.** $\int (x+a)^n\, dx = \frac{(x+a)^{n+1}}{n+1} + k$ **[Ans.: C]**

**46.** **[Ans.: A]**

**47.** $4 = 3t^4 + 5t^3 + 2t^2 + t + 4$

$\therefore \frac{dy}{dt} = 12t^3 + 15t^2 + 4t + 1$ $\quad (t = -1)$

$= 12(-1)^3 + 15(-1)^2 + 4(-1) + 1$

$= -12 + 15 - 4 + 1$

$= 0$ **[Ans.: A]**

**48.** $\int 5x^2 dx = 5\left(\frac{x^3}{3}\right) + c = \left(\frac{5}{3}\right)x^3 + k$ **[Ans.: D]**

**49.** $\int \frac{1}{\sqrt{x^5}} dx = \int x^{-5/2} dx = \frac{x^{-3/2}}{-3/2} + c$

$= -\frac{2}{3}x^{-3/2} + c$ **[Ans.: D]**

**50.** $y = (x - 1)(x + 1)$

$\therefore y = x^2 - 1$

$\therefore \frac{dy}{dx} = 2x$

$\therefore \frac{d^2y}{dx^2} = 2$ **[Ans.: A]**